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ANALYSIS OF RECOVERY ERRORS OF QOFDM SIGNALS

Valerii Kozlovskyi[1]
vvkzeos@gmail.com
Liudmyla Klobukova[1]
klp@nau.edu.ua
Volodymyr Lysechko[2]
volodymyr.lysechko@gmail.com
Galina Cherneva[3]
cherneva@vtu.bg

[1]National Aviation University, Kyiv, Lyubomyr Huzar avenue, 1,UKRAINE
[2]Ukrainian State University of Railway Transport, Kharkiv, Feuerbach Square 7, UKRAINE
Todor Kableshkov University of Transport, Sofia, Geo Milev Str. 158, BULGARIA

Keywords: *quasiorthogonal frequency division of channels, frequency collisions, subcarrier frequency, lag module, Gibbs pulsations.*

Abstract: *A statistical analysis of complex signals correlation properties formed on the basis of quasi-orthogonal access at subcarrier frequencies was carried out. A comparison of the influencing factors of the sampling frequency and the number of the decomposition series members was made using graphoanalytical methods. An analysis of the characteristics of the Gibbs pulsations appearance with different numbers of decomposition terms was carried out. The specified emissions are observed at all points of a jump-like change in signals and can be quite significant.*

A graphical analysis of the expansion in a series with the specified criteria for compliance with the conditions of Kotelnikov's theorem was carried out. He showed that choosing the highest possible sampling rate cannot be a key requirement for orthogonality. It was found that it is required to observe the integer ratios between the sampling frequency and the width of the main and side lobes of the signal spectrum.

The results of the study of the properties of such signals make it possible to optimize the process of selecting signal parameters. The consequence of this is the possibility of increasing the volume of signal ensembles with low interaction in the frequency domain. This makes it possible to increase the bandwidth of the cognitive radio network.

INTRODUCTION

The implementation of efficient communication systems with high data transfer rates requires the use of signals with a high modulation order. The principle of orthogonal frequency division multiplexing (OFDM) for data exchange at an efficient rate in various signal transmission environments, with minimal inter-channel and inter-symbol interference, has been widely used for several decades.

The main advantage of OFDM over modulation methods with a single carrier frequency is the extended possibilities of adaptation to changing conditions of the data transmission channel - electrical wire, wireless or optical. Factors of external influence include fast or slow fading in the propagation medium, multipath propagation, which causes frequency-selective fading, etc. The logical development of this method was the method of quasi-orthogonal frequency access QOFDM. The key element of quasi-orthogonal frequency access on subcarrier frequencies is the use of individual distribution of frequency subcarriers in different frequency plans of the ensemble in the common frequency band, which allows significantly increasing the subscriber capacity of the radio system and reducing the level of intersystem interference in the process of forming frequency plans.

The properties of ensembles of complex signals based on QOFDM have not been sufficiently studied, therefore, in order to develop recommendations for the implementation of complex signals based on QOFDM, an assessment of their intercorrelation properties was carried out, taking into account the different values of the number of subchannels, their width and the total spectrum width of the ensemble of frequency plans.

THE MAIN PART

The choice of QOFDM system parameters is related to ensuring operation in single-frequency radio and television broadcast networks, as well as the possibility of using gap fillers and dead zones in the area of broadcast coverage.

A method of quasi-orthogonal access at subcarrier frequencies was developed, which is based on the principle of zero orthogonality between frequency positions [1].

In order to assess the possibilities of using the frequency resource under the condition of using quasi-orthogonal access on subcarrier frequencies, it is necessary to study the degree of influence of intra-system interference when changing the width of subchannels between different frequency plans. A channel model was built in which the degree of mutual correlation between them changed for 4 values of the number of subchannels. The degree of similarity of the frequency plans was evaluated by calculating the mutual correlation function [3-5].

The mathematical mechanism is implemented on the basis of correlation analysis.

The greatest theoretical and practical interest are signal recovery errors after reception and processing by digital methods. A comparison of the influence factors of the sampling frequency and the number of members of the decomposition series is also of great interest.

Analysis of OFDM signal recovery errors

In telecommunications, received signals and time series are usually influenced by many complex factors and appear as multi-component non-stationary modes. In many situations, it is necessary to split these signals or time series into a finite number of monocomponents to represent the internal modes.

The greatest theoretical interest are signal recovery errors after reception and processing by digital methods. A comparison of the influence factors of the sampling frequency and the number of members of the decomposition series is also of great interest.

As noted in work [6], Kotelnikov's theorem is widely used in the theory and practice of electrical signal analysis, on the basis of which the corresponding series is formed. Members of the series are functions of the form $\operatorname{sinc} x = \sin x / x$, invariant to time shift.

Since the amplitude-frequency spectrum of a rectangular pulse is described by the function $\left|\operatorname{sinc}(x)\right| = \left|\sin(x)/x\right|$, expansion into the Kotelnikov series is the most natural and visual tool for forming a discrete signal, analyzing its parameters, and restoring it on the receiving side.

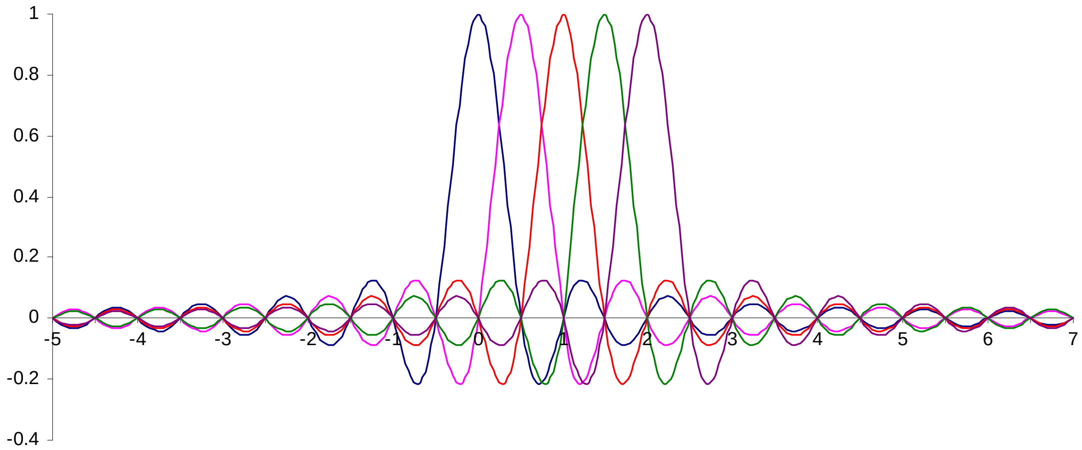

Fig. 1. Graph of the five terms of the rectangular pulse expansion series. Sampling frequency $f_d = 2/\tau_p$

When choosing the frequency by discretization according to Kotelnikov's theorem, the condition of orthogonality is observed, and the signal recovery error (lag recovery) becomes the minimum possible. The magnitude of the error depends only on the number of members of the decomposition. In fig. 2. graphs of the original signal (dashed line) and the reconstructed signal (solid line) are shown, and in fig. 3 - the graph of the recovery lag module.

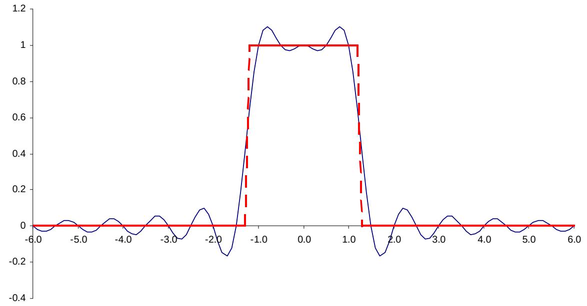

Fig. 2. Graphs of the original signal (rectangular pulse) and the restored signal after its processing on the receiving side

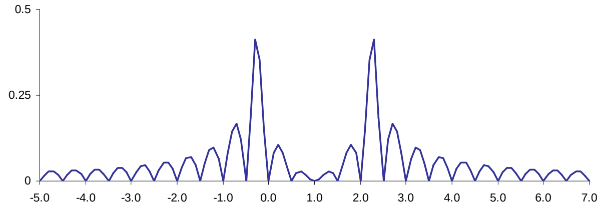

Fig. 3. Graph of the recovery lag module

After limiting the number of members of the expansion series, Gibbs pulsations occur [7]. They have the following characteristics:

- the contour of the pulsation attenuation is close to the contour of the amplitude-frequency spectrum of the output signal;
- the frequency of pulsations is close to the frequency of the first discarded member of the series;
- the amplitude of the main petal of the pulsations does not change when the number of expansion terms increases. This can be practically verified by analyzing pulsations with different numbers of terms of the decomposition.

The function of the recovery lag module is as follows:

$$\varphi_{div}(t) = \left|g_{\varepsilon}(t) - g(t)\right|, \quad g(k) = g(k\Delta t_{discr}), 1 \le k \le 5, \tag{1}$$

where $g(t)$, $g_{\varepsilon}(t)$ are the original signal and the restored signal;
$g(k\Delta t_{discr})$, $1 \le k \le 5$ – a set of five discrete readings of a continuous signal.

The Euclidean norm [8] of the distance between $g(t)$ and $g_{\varepsilon}(t)$ is determined by the expression:

$$(2) \qquad \delta_{g\varepsilon} = \sqrt{\int_{-\infty}^{\infty} \left[g_{\varepsilon}(t) - g(t)\right]^2 dt},$$

and the relative square of the norm of the distance between $g(t)$ and $g_{\varepsilon}(t)$ - by the expression:

$$(3) \qquad \sigma_{\varepsilon}^2 = \delta_{g\varepsilon}^2 / \|g(t)\|^2, \ -t_{\max} \le t \le t_{\max}, \ |t_{\max} < \infty|.$$

Unfortunately, in radio communication systems, multipath propagation is almost inevitable, which leads to distortions of the received signal. To eliminate such interference, it is necessary to choose a guard interval whose duration is longer than the maximum propagation delay of the transmission channel. In this way, it is possible to compensate for both the interference between subcarriers and between adjacent transmission blocks (intersymbol interference). To reduce out-of-band radiation, signal filtering with a “raised cosine” type window is used.

When the number of members of the decomposition series increases, it should be expected that the reconstructed signal will more accurately repeat the original signal. Fig. 4 shows the graphs of the original signal (rectangular pulse of unit length) and the restored signal when expanded into a series with 21 terms. Fig. 5 shows a graph of signal recovery inconsistencies.

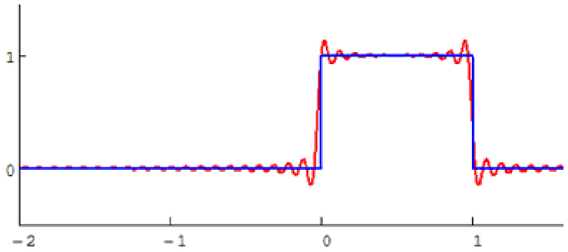

Fig. 4. Graphs of the original and restored signals

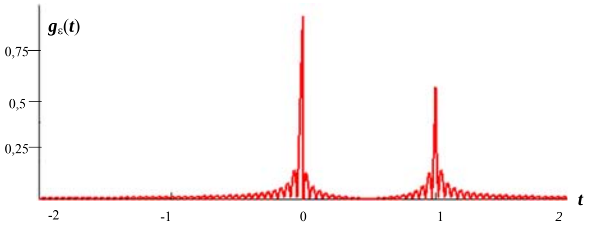

Fig. 5. The graph of recovery disconnections

When performing calculations according to formulas (1 – 3), it was established that during signal restoration, the mean square of the Euclidean distance between the original and restored signals rapidly decreases with an increase in the sampling frequency (Fig. 6).

It would seem that the higher the sampling rate, the better the accuracy of the signal recovery, but other influencing factors must be taken into account here.

First, when the sampling frequency increases, the requirements for the speed of digital signal processing devices increase. Secondly, the harmful influence of interference of various natures that penetrates into broadband systems is growing [9, 10]. Therefore, the final decision regarding the choice of signal sampling frequency in systems with many subcarriers should be made on the basis of a systematic approach, taking into account conflicting conditions and making compromise decisions.

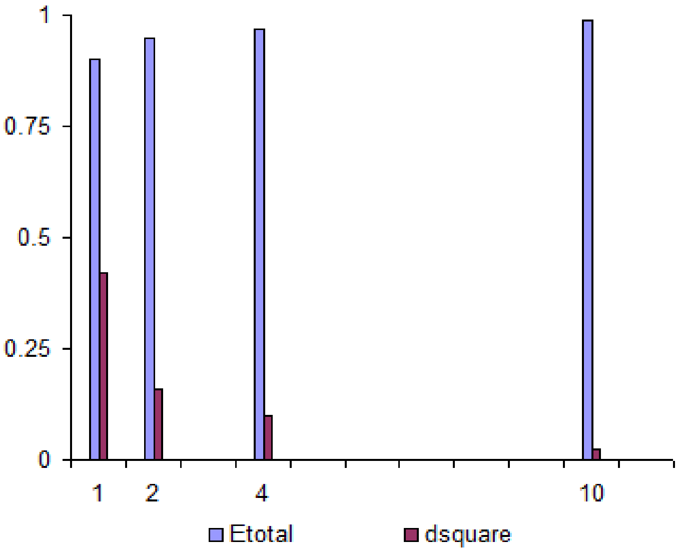

Fig. 6. Dependence of the square of the Euclidean distance d_{square} between the original and reconstructed signals on the sampling frequency

In conclusion, we will investigate the expansion into a series of five terms, when the conditions of Kotelnikov's theorem are not met (Figs. 7-9).

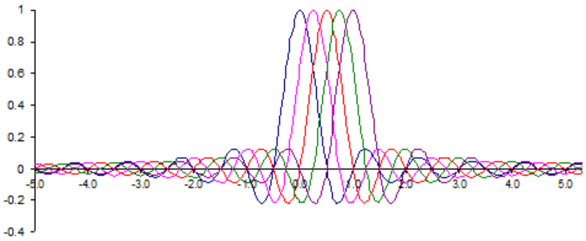

Fig. 7. Graph of five terms of the expansion series of a rectangular pulse. Sampling frequency $f_d = 1/\tau_p$

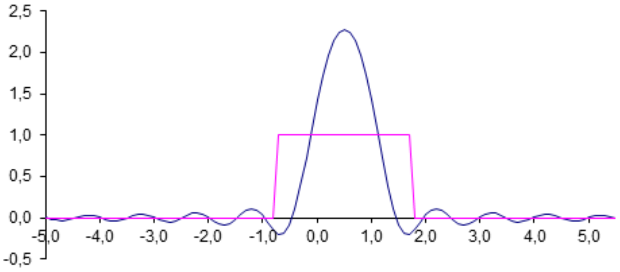

Fig. 8. Graphs of the original signal (rectangular pulse) and the restored signal after its processing on the receiving side. Sampling frequency $f_d = 1/\tau_p$

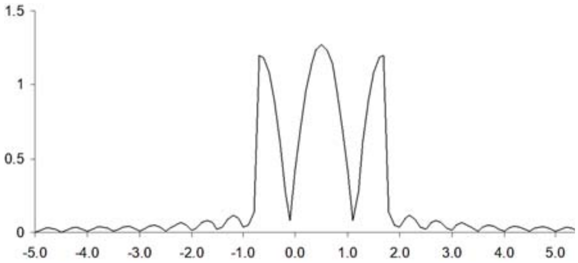

Fig. 9. Signal recovery lag graph

Research results indicate that when the conditions of orthogonalization of the system of basis functions are violated, the Euclidean norm of the distance between $g(t)$ and $g_\varepsilon(t)$ (expression (2)) becomes unacceptably large, and a simple increase in the number of terms of the series expansion does not give a noticeable improvement.

This is especially evident in the presence of discontinuities (at least of the first kind) in the signal functions, such as, for example, rectangular pulses. The steepness of the dips "washes out" because it cannot be greater than the steepness of the last retained harmonic of the series:

$$(4)\qquad g(t)=\sum_{n=-\infty}^{\infty} g\left(\frac{n}{2f_{g\max}}\right)\frac{\sin\left[2\pi f_{g\max}\left(t-n/2\pi f_{g\max}\right)\right]}{2\pi f_{g\max}\left(t-n/2\pi f_{g\max}\right)}=\sum_{n=-\infty}^{\infty} g(n\Delta t)\varphi_n(t).$$

Emissions and pulsations with a frequency close to the frequency of the first rejected term of the series appear on both sides of the "blurry" differences.

Some improvement can be given by increasing the number of terms of the expansion series, although no guarantees of a monotonous improvement of the situation with recovery errors can be given in principle. Aig. 10 and 11 show the graphs of the expansion in a series with 11 members and 21 members.

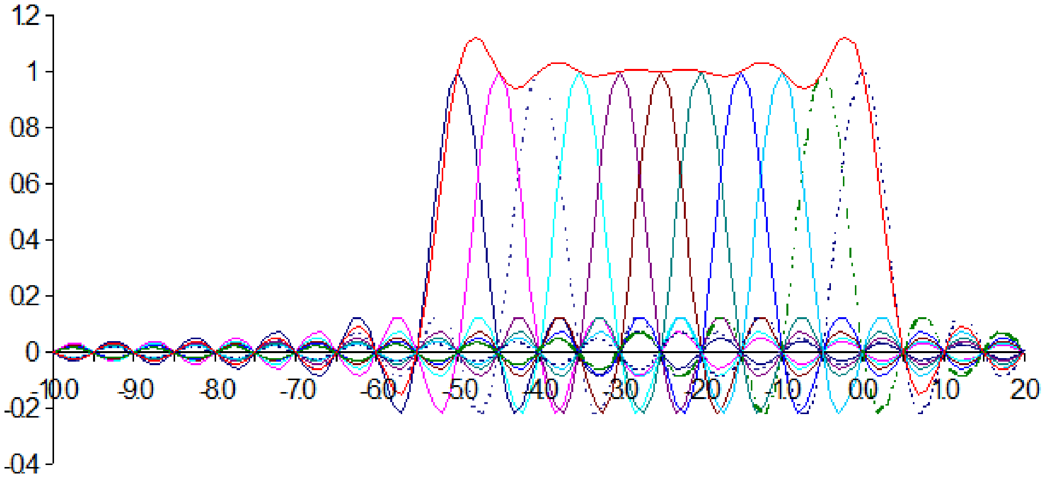

Fig. 10. Graphs of 11 decomposition members

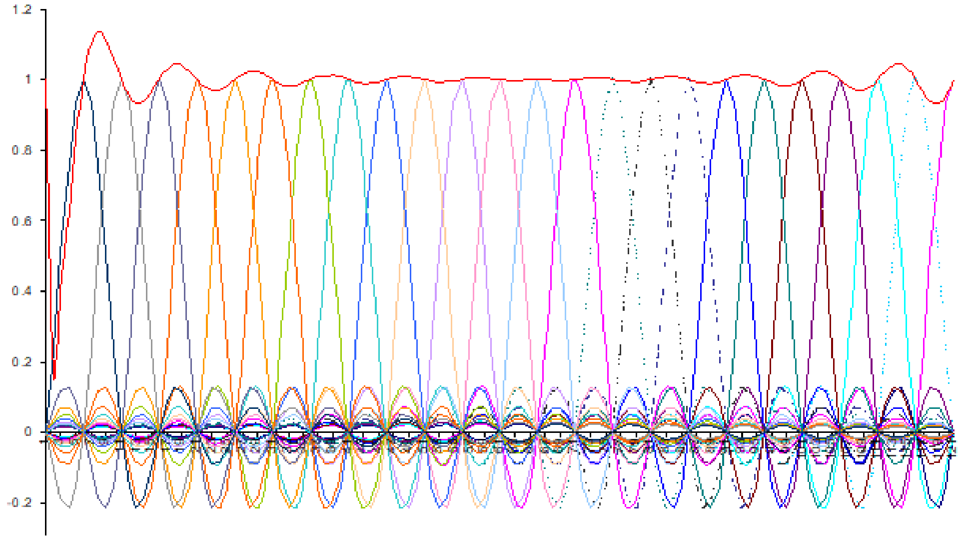

Fig. 11. Graphs of 21 decomposition members

If the frequency shift between adjacent members of the series is chosen equal to $\Delta f=1/\tau_p$, the condition of Kotelnikov's theorem is not met. Accordingly, the orthogonality of the decomposition is also not observed, although the invariance to the time shift, due to the properties of the basis functions of the decomposition, is preserved. Since the orthogonality condition is not met, the side lobes of the partial spectra intersect at points where their values are arbitrary and not equal to zero.

In full compliance with the violation of orthogonality conditions, the summation of the terms of the series gives unsatisfactory results in terms of signal recovery errors.

Concluding the graphic analysis of series expansion with different strictness of compliance with the conditions of Kotelnikov's theorem, we note that the key requirement is not so much the selection of the highest possible sampling frequency, but the observance of integer ratios between the sampling frequency and the width of the main and side lobes of the signal spectrum.

CONCLUSIONS

A statistical analysis of correlation properties of complex signals formed on the basis of quasi-orthogonal access at subcarrier frequencies was carried out. The results of the study of the properties of such signals make it possible to optimize the process of selecting signal parameters, which ensure an increase in the volume of signal ensembles with low interaction in the frequency domain.

Signal recovery errors after reception and processing by digital methods are of not only theoretical, but also practical interest. A comparison of the influence factors of the sampling frequency and the number of members of the decomposition series is also of great interest. This comparison was made by the grapho-analytical method.

The method of determining frequency positions makes it possible to simplify the process of forming frequency plans and reduce the level of intra-system interference that occurs when many users use the same frequency bands simultaneously in cognitive radio systems. This makes it possible to increase the bandwidth of the cognitive radio network.

Graphical analysis of series decomposition with different strictness of compliance with the conditions of Kotelnikov's theorem showed that the key requirement is not so much the selection of the highest possible sampling frequency, but the observance of integer ratios between the sampling frequency and the width of the main and side lobes of the signal spectrum.

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