Mechanics
Transport
Communications
Academic journal

ISSN 1312-3823
issue 2, 2007
article № 0092
http://www.mtc-aj.com

STRESS AND STRAIN WAVES AT IMPACTS OF WAGGONS

Dragan Petrovic, Ranko Rakanovic, Zlatan Soskic
petrovic.d@ maskv.edu.yu, rakanovic.r@ maskv.edu.yu, soskic.z@maskv.edu.yu

Faculty of Mechanical Engineering Kraljevo,
Dositejeva 19, SERBIA

Key word: *Wave, Impact, Waggon*

Abstract: *This paper presents theoretical and experimental analysis of wave phenomena at impact of waggons. Theoretical considerations have been realized on an idealized beam model, and experimental results refer to test of waggon Zagkks for transport of liquid petroleum gas.*

1. BEHAVIOUR OF ELASTIC BODIES AT IMPACT

At longitudinal impacts, when the structure members are very quickly deformed, complex physical phenomena occur, such as: changes of rheological properties of the material, temperature and chemical changes, etc. During these phenomena, the behaviour of the structure can be completely different from its behaviour at static loading. The structure fails in getting displacements which correspond to fast changes of loads. Such delay can cause abrupt deformation of the structure.

Fast changes in stresses and strains caused by impact cannot be precisely defined without considering wave processes. Therefore, where it is possible, the wave character of propagation of deformations is observed in theoretical research of behaviour of elastic bodies at impact. However, in railway vehicles, where the geometry of the carrying structure is complex, and speeds of impact are not so great, a model of elastic body neglecting some phenomena can be formed. In that way, local effects which refer to the three-axis stress state is avoided. This postulation defines impact by a certain speed of a cross-section of the member or the shell and the ratio between masses of the observed elements and load.

Consideration of impact phenomena is, in this way, different from the case where the change of several physical factors is present and where changes of the structure of the material are dominant. Most real structures subjected to impact action can be treated in this way.

In that case, equations of motion [1,2] have the form:

$$(1)\quad \begin{aligned} &(\lambda+G)\frac{\partial\varepsilon_v}{\partial x}+G\nabla^2u+F_x-\rho\frac{\partial^2u}{\partial t^2}=0\\ &(\lambda+G)\frac{\partial\varepsilon_v}{\partial y}+G\nabla^2v+F_y-\rho\frac{\partial^2v}{\partial t^2}=0\\ &(\lambda+G)\frac{\partial\varepsilon_v}{\partial z}+G\nabla^2w+F_z-\rho\frac{\partial^2w}{\partial t^2}=0 \end{aligned}$$

where:

λ, G, ρ – constants of material,

ε_v – volume deformation,

u, v, w – displacements in x, y and z directions,

F_x, F_y, F_z – external volume forces,

t – time

∇^2 – Laplace operator.

2. PROPAGATION OF WAVES IN ELASTIC CONTINUUM

Behaviour of an elastic body loaded with forces which do not change in time belongs to the field of statics. These problems can include the case where the change of load in time is slow, i.e. quasistatic. If changes of load in time are faster, as in the case of impact loads, then the problems are transferred to the field of dynamics. Then it is necessary to replace the equations of static equilibrium of an elastic body by equations of motion. In this case, action of dynamic (impact) load is not immediately transmitted to all points of the body. Waves of stresses and strains start to propagate from the loaded surface and they have finite speed of propagation. Here, as in the familiar case of propagation of sound in the air, a certain point will be incited only when a wave reaches it. In an elastic body, there is not only one wave, but several types of waves and they have different speeds of propagation.

2.1 Longitudinal and cross waves in isotropic elastic continuum

If a certain point of the elastic continuum is incited, waves will start to propagate from that point to all sides. It can be taken that, at a distance from the centre of incitation, all particles will move in parallel with the direction of propagation of waves (longitudinal waves) or normally to that direction (cross or transversal waves).

Under the assumption that, in the existence of waves, the volume deformation is equal to zero, i.e. that deformation consists of sliding and rotating only, equations (1) obtain the form [1, 2]:

$$(2)\quad \begin{aligned} &\frac{G}{\rho}\nabla^2u=\frac{\partial^2u}{\partial t^2}\Rightarrow\quad \frac{\partial^2u}{\partial t^2}=c_2^2\,\frac{\partial^2u}{\partial x^2}\\ &\frac{G}{\rho}\nabla^2v=\frac{\partial^2v}{\partial t^2}\Rightarrow\quad \frac{\partial^2v}{\partial t^2}=c_2^2\,\frac{\partial^2v}{\partial y^2}\\ &\frac{G}{\rho}\nabla^2w=\frac{\partial^2w}{\partial t^2}\Rightarrow\quad \frac{\partial^2w}{\partial t^2}=c_2^2\,\frac{\partial^2w}{\partial z^2} \end{aligned}$$

The previously obtained wave equations represent cross waves. The value c_2 is the speed of cross waves in the elastic continuum and it is determined by the expression:

(3) $$c_2 = \sqrt{\frac{G}{\rho}}$$

The essential difference between solving dynamic and static problems is that the boundary conditions should be added by initial conditions, i.e. displacements and speeds of points at a certain initial moment of the time t_o should be defined.

Let us consider the case when the deformations occurring due to the action of waves do not contain rotation. Rotation of an elementary part is determined by the equations:

(4) $$\omega_x = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right), \quad \omega_y = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right), \quad \omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

On the basis of the condition that deformation should not contain rotation, the following can be written:

(5) $$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0, \quad \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} = 0, \quad \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} = 0$$

If these conditions are satisfied, displacements u, v and w can be expressed by a function φ in the following way:

(6) $$u = \frac{\partial \varphi}{\partial x}, \quad v = \frac{\partial \varphi}{\partial y}, \quad w = \frac{\partial \varphi}{\partial z}$$

Then:

(7) $$\varepsilon_v = \nabla^2 \varphi, \quad \frac{\partial \varepsilon_v}{\partial x} = \frac{\partial}{\partial x} \nabla^2 \varphi = \nabla^2 u$$

By replacing these equations in the equations of motion, the following is obtained:

(8) $$(\lambda + G)\nabla^2 u - \rho \frac{\partial^2 u}{\partial t^2} = 0 \Rightarrow \quad \frac{\partial^2 u}{\partial t^2} = c_1^2 \frac{\partial^2 u}{\partial x^2}$$
$$(\lambda + G)\nabla^2 v - \rho \frac{\partial^2 v}{\partial t^2} = 0 \Rightarrow \quad \frac{\partial^2 v}{\partial t^2} = c_1^2 \frac{\partial^2 v}{\partial y^2}$$
$$(\lambda + G)\nabla^2 w - \rho \frac{\partial^2 w}{\partial t^2} = 0 \Rightarrow \quad \frac{\partial^2 w}{\partial t^2} = c_1^2 \frac{\partial^2 w}{\partial z^2}$$

The waves determined by these equations are called longitudinal waves or propagation waves.

The speed of propagation of longitudinal waves is given by the expression:

(9) $$c_1 = \sqrt{\frac{\lambda + 2G}{\rho}}$$

From the equations (3) and (9), it can be seen that waves in the elastic continuum can propagate at two different speeds. In propagation waves, the direction of motion of particles coincides with the direction of propagation of waves, while cross waves, which occur due to rotating and sliding, propagate normally to that direction.

Let us first consider longitudinal waves. If the axis *x* is in the direction of propagation of waves, then *v=w=0*, so that the displacement *u* is a function of the coordinate *x*. In that case, the wave equation of the system (8) is determined by:

(10) $$\frac{\partial^2 u}{\partial t^2} = c_1^2 \frac{\partial^2 u}{\partial x^2}$$

Every function $f(x+c_1t)$ can be a solution to the previous equation. Also, every function $f_1(x-c_1t)$ is a solution to that equation, so that it is possible to write the general solution in the form [3]:

(11) $u=f(x+c_1t)+f_1(x-c_1t)$

This solution has the following physical interpretation. Let us consider the second member of the previous equation. At every moment of the time *t,* that member appears with a function of only one variable and can be represented by a curve whose shape depends on the function f_1. Through the time interval Δt, the argument of the function f_1 obtains the form $x-c_1(t+\Delta t)$. The value of the function f_1 remains unchanged if, simultaneously with the increase of the time *t* by the value Δt, the abscissa is increased by the value $\Delta x=c_1\Delta t$. It means that the wave function formed at a point of the time *t* can also be used in the time $t+\Delta t$ if it is moved along the axis *x* at the distance $\Delta x=c_1\Delta t$. The first member of the equation (11) has the same behaviour, but this wave propagates in the opposite direction. In that way, the general solution to the equation (11) can be represented by two waves moving along the axis *x* in two opposite directions at the constant speed c_1. This speed can also be expressed through the module of elasticity *E* and the Poisson's coefficient ν:

(12) $$c_1 = \sqrt{\frac{E(1-\nu)}{(1+\nu)(1-2\nu)\rho}}$$

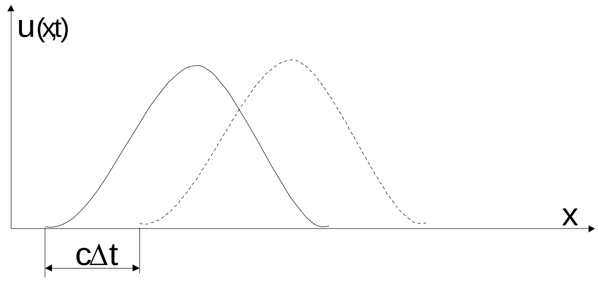


Figure 1. Propagation of waves in the elastic continuum

Considering the physical motion of waves given by the function $f_1(x-c_1t)$, the following expression for the speed of particles is obtained:

(13) $$\dot{u} = \frac{\partial u}{\partial t} = -c_1 f_1'(x - c_1 t)$$

Using the expression for the stress σ_x, as well as the expression for the speed of particles $\dot{u}$ (13), the stress in the direction of propagation of waves is obtained:

(14) $$\sigma_x = -\rho c_1 \dot{u}$$

If the returning motion of waves represented by the first member in the equation (11) $f(x+c_1t)$ is observed, the minus sign in the equations (13) and (14) would be replaced with the plus sign.

The functions f_1 and f for each separate case are determined from the initial conditions at the moment $t=0$, where

$$(u)_{t=0}=f(x)+f_1(x),$$

(15) $$\left(\frac{\partial u}{\partial t}\right)_{t=0} = c_1\left[f'(x) - f_1'(x)\right]$$

If the initial speed is equal to zero, and the initial displacement is determined by the function: $(u)_{t=0}=F(x)$, the previously mentioned conditions will be fulfilled if $f(x)=f_1(x)=\frac{1}{2}F(x)$. In that case, the initial displacement is divided in two halves propagating wavily in two opposite directions.

2.2 Beam at longitudinal impact

This section presents main dependencies referring to behaviour of beams at longitudinal impact. Let us observe the beam with the mass m_2 whose one end is subjected to the action of impact load originating from the absolutely rigid body with the mass m_1, which moves with the speed v_1 until it meets the beam. The beam at the end $x=\ell$ can be stationary or freely supported and it is in the state of rest, [4, 7, 11, 24].

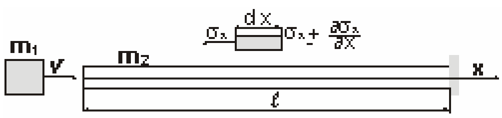


Figure 2. Beam at longitudinal impact

At the beginning of impact, the beam is compressed, so that the initial speed v_1 of the mass m_1 is impulsely changed until the speed of displacement of the beam end which undergoes the impact $\dot{u} = \partial u/\partial t$. This leads to fast occurrence of deformations $\varepsilon = \partial u/\partial x$, that is the stresses $\sigma_x = E \cdot \varepsilon$.

Let us consider only propagation of waves along the direction of impact, neglecting the process of oscillation in the body performing the impact.

On the basis of the expression (8), the differential equation of displacement of the beam along the axis x has the form:

(16) $$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2},$$

where the speed of propagation of waves in the beam is:

(17) $$c = \sqrt{\frac{E \cdot g}{\gamma}} = \sqrt{\frac{E}{\rho}}$$

At the moment of reaching the maximum displacement in the beam u_{max}, the mass m_1 will be in the state of rest. If the kinetic energy before the impact is $E_{k,o}$ and the maximum potential energy of the system is $E_{p,max}$, then, on the basis of the law of conservation of energy, it can be written:

(18) $$E_{k,o}=E_{p,max}=\frac{1}{2}m_1 v_1^2 = \frac{1}{2}EA\ell\left(\frac{u_{max}}{\ell}\right)^2$$; or

$$\varepsilon_{max} = -\frac{u_{max}}{\ell} = -v_1\sqrt{\frac{m_1}{EA\ell}} = -\frac{v_1}{c}\sqrt{\frac{m_1}{m_2}} = -\frac{v_1}{c}\sqrt{\kappa}$$

Here, κ is the ratio between the mass of load and the mass of the beam.

If the impact speed v is above a certain limit dependent on mechanical properties of the beam, a permanent deformation can occur in it although the mass of the impact body is small.

In the case of propagation of waves in the beam whose end $x=\ell$ is stationary, the solution to the equation (16) has the form (11) and must satisfy the following conditions:

$u(x,0)=0,$

$\frac{\partial u}{\partial t}(x,0) = v_1$; pri $x=0$;

(19) $\frac{\partial u}{\partial t}(x,0) = 0$; when $x>0$

$\frac{\partial^2 u}{\partial t^2}(0,t) = \frac{c^2}{\kappa\ell}\frac{\partial u}{\partial x}(0,t)$; $u(\ell,t)=0$

Time is measured from the moment of impact.

Let us assume that the instrument for registration of displacements and deformations, determined by the function f_1, moves with the speed c from the free end to the supported end. In that case, at the points where the instrument is positioned, we shall have that $x=ct$, f_1=const, i.e. indication of the instrument will not change. Hence, it follows that the function f_1 determines the wave deformation, which propagates along the beam in the direction from the point of impact toward the supported end, and c is the speed of propagation of the wave front equal to the speed of sound in the beam.

The sense of the function f is the wave reflecting from the stationary end. The local speed of the beam particles ($\dot{u}$) and deformations (ε) is determined by appropriate derivations:

(20)
$$\left.\begin{aligned} \dot{u} &= \frac{\partial u}{\partial t} = cf_1'(ct-x) + cf'(ct+x) \\ \varepsilon &= \frac{\partial u}{\partial x} = -f_1'(ct-x) + f'(ct+x) \end{aligned}\right\}$$

Let us consider the initial period of deformations $0 \le t \le \ell/c$. If f=0 and x=0, the equation for determination of displacement of the loaded end is obtained:

(21) $f_1''(t^*) + \frac{1}{\kappa\ell}f_1'(t^*) = 0$

where $t^*=ct$

By using the limiting conditions, the following expressions for the speed of displacement of the movable end of the beam and for the corresponding deformation are obtained:

(22) $$\begin{rcases} \dot{u}(0,t)=\dfrac{\partial u(0,t)}{\partial t}=v_1\cdot e^{-\frac{t^*}{\kappa\ell}} \\ \varepsilon(0,t)=-\dfrac{v_1}{c}\cdot e^{-\frac{t^*}{\kappa\ell}} \end{rcases}$$

Hence, it follows that at the moment of impact the members of the beginning of the beam, which are subjected to impact, obtain the deformation equal to the ratio of the local speed of the initial point of the beam and the speed of sound in the beam.

The displacement of the end point of the beam is determined by the expression:

(23) $$u(0,t)=v_1\frac{\kappa\ell}{c}(1-e^{-\frac{t^*}{\kappa\ell}})$$

If the mass of the body which performs impact is considerably greater than the mass of the beam, it can be considered that $\kappa=\infty$, and at the speed $v_1=const.$ from the equation (22), it follows:

$u(0,t)=v_1 t$

For the analysis of the time period $\ell\le ct\le 2\ell$, it is necessary to determine the function f and limiting conditions at the stationary end. In that way, direct integration of the equation (16) results in functions whose form is changed upon running out of the period which is equal to the period of passing of the elastic wave along the beam. In the time period $t=2\ell/c$, the pressure wave returns to the beam beginning, which is in contact with the body. The speed of the body cannot be abruptly changed, so that the wave will reflect as if from the fixed end, and thus be doubled.

The characteristic curve of the beam deformation at longitudinal impact has the exponential form which decreases in time and after the period $2\ell/c$ has a rise. The value of the exponent is determined by the ratio between the masses of the body and the beam κ. The length of duration of the contact depends on the speed of members at impact v_1 and the ratio of masses κ. The contact stops at the moment when deformation of the beam beginning is equal to zero, which corresponds to passing through the equilibrium state.

If the other end of the beam is free, instead of the condition $u=0$ when $x=\ell$, it is necessary to introduce the condition $\partial u/\partial x=0$.

3. SPEEDS OF WAGGONS AT IMPACT

The impact of two waggons can be observed as the impact of two beams (Fig. 3) moving at the speeds v_1 and v_2 $(v_1>v_2)$.

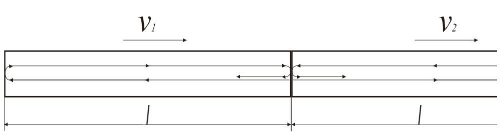


Figure 3. Impact of two beams

At the moment of impact, two identical pressure waves start moving along both beams. In order to obtain equal absolute speeds of particles of both beams over the contiguous surface, the values of those speeds must be equal to $(v_1-v_2)/2$. After the time interval *l/c*, pressure waves reach free ends of the beams. At this moment, both beams are in the state of uniform pressure and the absolute speeds of all particles of the beams are:

(24) $$v_1 - \frac{v_1 - v_2}{2} = v_2 + \frac{v_1 - v_2}{2} = \frac{v_1 + v_2}{2}$$

Pressure waves will then reflect from the free end, and at the moment $2\ell/c$, when these waves reach the contiguous surface of both beams, their speeds become:

(25) $$\frac{v_1 + v_2}{2} - \frac{v_1 - v_2}{2} = v_2$$ the speed of the first beam

(26) $$\frac{v_1 + v_2}{2} + \frac{v_1 - v_2}{2} = v_1$$ the speed of the first beam

i.e., the beams change their speeds during the impact.

The previously exposed theory of impact is based on several assumptions, such as, it is the impact of two homogeneous beams, the contact occurs over the whole surface of the beam, at the same moment, etc. In practice, such a case is rare and that is why the results of theoretical and experimental research do not agree. However, the knowledge of principles of occurrence and propagation of waves can help us in the analysis of experimentally obtained results of tests of real structures.

4. ANALYSIS OF EXPERIMENTAL RESULTS OF IMPACT OF WAGGONS

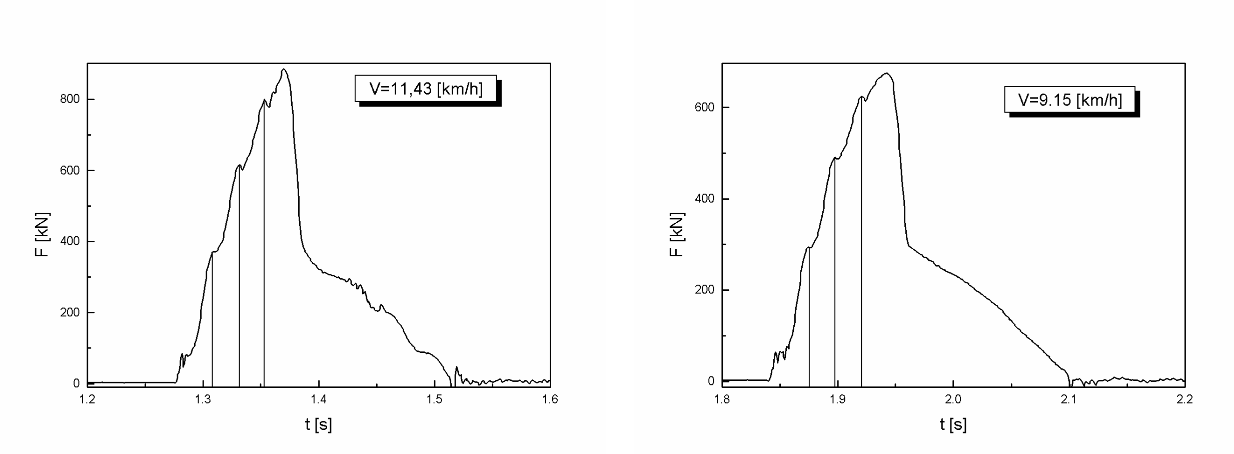
The Centre for Railway Vehicles at the Faculty of Mechanical Engineering in Kraljevo performs tests of wagons, where testing of waggons at impact is one of obligatory tests.

During the impact of real waggon structures (Fig. 4), due to the complicated carrying structure, it is impossible to use only theory, for the time being, to determine precisely all parameters occurring at that. However, theoretical considerations can help us in the analysis of experimentally recorded data because the character of the phenomenon is the same.

Figure 4. Tank-wagon

Figure 5. a) a b) clearly shows effects of wavy motion, i.e. the time necessary for the wave to pass from the buffer to the end of the waggon and back. The experimentally determined time for this is between 21 and 24 ms and it is somewhat longer than in the case when two homogeneous members of the same length would be at impact. The cause of this "delay" of wave is explained by the non-homogeneous structure which is interweaved with elements of different characteristics, then by the shape of the contiguous surfaces participating in the impact, etc.

It can be indirectly concluded that the transducers, which record the impact force, have a satisfactory dynamic characteristic because they are able to record a phenomena which lasts more than ten times less than the time of impact duration. In the transducers which do not have a satisfactory dynamic characteristic, the curve would have a continual increase (without rises), and in that case there would appear an error in recording the maximum impact force.



a) b)

Figure 5. Change of force at the buffer at impact of wagons

5. LITERATURE

[1] PETROVIC D., Stability of Waggon Carrying Structure at Impact, Ph. D. thesis, Faculty of Mechanical Engineering Kraljevo, 2000.

[2] PETROVIC D., Dynamics of Impact of Waggons, Zaduzbina Andrejevic, Belgrade, 2001.

[3] GOLDSMITH W., Impact, The Theory and Physical Behaviour of Colliding Solids, London 1965.